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ABSTRACT

This handbook is part of a series of three, corresponding to the three grades (4, 6, and 8) at which Mastery Tests are administered. This publication was written as a resource for teachers developing mathematics programs for students in grades 4, 5, and 6. The Mastery Test in Mathematics at Grade 6 assesses student performance for 36 instructional objectives. The following information is provided for each objective: (1) Appropriate Materials (manipulative materials to use for exploring the concept); (2) Enabling Skills and Activities (a description of prerequisite skills and activities); (3) Sample Lessons (student activities that build toward mastery); (4) Teacher Resource Materials; and (5) Mathematics Objectives and Sample Test Items. The instructional objectives are grouped into four categories: (1) conceptual understandings; (2) computational skills; (3) problem solving and applications; and (4) measurement and geometry. Two appendixes are included: Appendix A contains the sample test items and objectives; Appendix B is a list of commercial suppliers of mathematics manipulatives. (RH)

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Handbook for Teachers

GRADE 6 MASTERY TEST IN MATHEMATICS

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SE 048 310

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Handbook for Teachers

GRADE 6 MASTERY TEST IN MATHEMATICS

This handbook was prepared by Betsy Y. Carter, mathematics consultant with the Connecticut State Department of Education.

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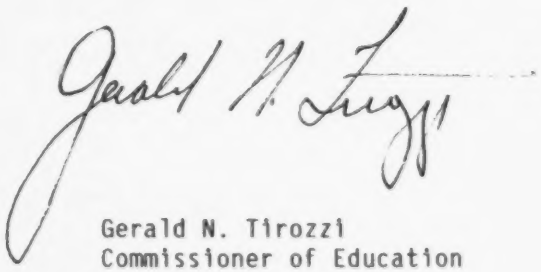
This *Handbook for Teachers, Grade 6 Mastery Test in Mathematics* represents another step toward our goal of insuring that all students master the mathematical concepts and skills required to become productive citizens. It relates to the achievement of three of the seven Board of Education goals--to improve the quality of instruction and curriculum, to improve effectiveness of teachers and teaching, and to insure equity for all children.

The 6th grade teachers' handbook is part of a series of three, corresponding to the three grades--4, 6, and 8--at which Mastery Tests are administered. The grade 4 handbook in mathematics was published in 1986; handbooks for teachers of grades 6 and 8 are being released in 1987.

The primary focus of this testing program is improvement in the achievement of Connecticut's students. Once teachers and administrators have reviewed class, school, and district test results, the task of using these results to improve student mastery of basic skills moves directly into the classroom. It is there--in thousands of Connecticut classrooms--where high quality teaching translates test results into meaningful instructional activities.

The *Handbook for Teachers, Grade 6 Mastery Test in Mathematics* includes specific instructional strategies and sample lessons that are keyed to each of the test objectives. You will also find sample test items and resource lists in the handbook.

I urge you to review carefully the Mastery Test results for your class and your school and to use this handbook as a resource for planning lessons and classroom activities that meet your students' needs for mathematics instruction.



Gerald N. Tirozzi
Commissioner of Education

The *Handbook for Teachers, Grade 6 Mastery Test in Mathematics* is written to serve as a resource for teachers as they develop a meaningful mathematics program for students in Grades 4, 5, and 6. The Mastery Test in Mathematics at Grade 6 assesses student performance on 36 objectives. However, mathematics instruction should NOT be reduced to providing lessons that are planned to meet only those objectives. Rather, instruction aimed at mastery of the objectives should be integrated into a mathematics program that has as its primary goal the understanding of important mathematics concepts and the use of mathematical ideas to solve problems.

The Mastery Test objectives were chosen as significant benchmarks of growth. The limitations of a multiple-choice, paper-and-pencil test have inevitably influenced the final list of objectives. Test items most frequently assess skills at a symbolic, or abstract, level; a few items include pictorial information. Teachers are urged to continue to assess student development of concepts through classroom observation of students as they work with manipulative materials, collect and organize information, and solve problems.

The role of this handbook is to place the objectives in the perspective of a mathematics curriculum that is based on the way children develop mathematical skills and concepts. This handbook also is designed to assist teachers in providing a mathematics program that continually moves through a sequence of concrete, pictorial, and abstract experiences as concepts are explored and objectives are mastered.

The ideas, activities, and sample lessons found in this handbook were designed to be a part of a mathematics curriculum that has problem solving as the central focus. Significant strands in the curriculum, which deserve equal time in the classroom, include:

- Organizing Information

  Classification
  Patterns
  Graphs and tables
  Probability
  Statistics
  Estimation

- Spatial Relationships

  Geometry
  Measurement
  Estimation

- Number Sense

  Order
  Place value
  Whole numbers, fractions, decimals, integers
  Operations and numbers
  Estimation
  Mental computation
  Computation with calculators

Handbook Components

In the handbook, the mathematics Mastery Test objectives are organized in clusters. The objectives in each cluster share the same underlying mathematical concepts.

- **Appropriate Materials.** Exploration of concepts through the daily use of manipulative materials promotes understanding. A listing of useful manipulative materials is included for each cluster of test objectives.

- **Enabling Skills and Activities.** Students should NOT be asked to practice examples at the same level of difficulty as the test items without experiences at earlier stages. Therefore, this handbook provides a description of prerequisite skills and activities suitable earlier in the sequence of instruction.

- **Sample Lessons.** Included in the sample lessons are ideas that directly involve students in activities that build toward mastery. Each lesson has been designed to develop or reinforce several skills and concepts. This presents an opportunity for students to engage in the process of making connections and solving problems in the various strands of the curriculum. Rather than teaching each skill in isolation, teachers are urged to explore ideas together so that students may begin to discover the structure of mathematics.

- **Teacher Resource Materials.** A list of teacher resource materials that can aid in the development of additional activities is provided for each cluster of objectives. The bibliography contains complete information about the resource materials, as well as citations for other professional research and reference materials.

- **Mathematics Objectives and Sample Test Items.** Appendix A contains a sample test item for each objective.

- **Suppliers.** Appendix B is a list of suppliers of commercial mathematics manipulatives.

CONCEPTUAL UNDERSTANDINGS

	Objective		Page
I. COMPARING NUMBERS	1	Order whole numbers less than 100,000.	7
II. PLACE VALUE	2	Identify the value of a digit in whole numbers less than 100,000 and rewrite whole numbers using expanded notation.	10
	3	Rename whole numbers by regrouping 1000's, 100's, 10's, and 1's.	10
	4	Round whole numbers less than 100,000 to the nearest 1000.	10
	5	Multiply and divide multiples of 10 and 100 by 10 and 100	10
III. EQUIVALENT FRACTIONS	6	Identify equivalent fractions and mixed numbers using pictures.	15
	7	Identify equivalent fractions and mixed numbers.	15
IV. DECIMAL CONCEPTS	8	Identify decimals (.01 to 2.99) from pictorial representations.	21
V. PATTERNS	9	Extend patterns involving numbers and attributes.	25

* The test objectives are reorganized into clusters of related concepts. The numbering of objectives as listed on the Mastery Test Student Report has been maintained to serve as a cross-reference to the test. However, for instructional and mathematical clarity, a few objectives have been addressed out of numerical sequence.

COMPUTATIONAL SKILLS

		Objective		Page
VI.	ESTIMATION	10	Identify an appropriate procedure for making estimates for whole number computations.	31
		19	Estimate sums and differences of whole numbers and money amounts.	31
		20	Estimate products and quotients of whole numbers and money amounts (1-digit factor and 1-digit whole number divisor).	31
VII.	ADDITION AND SUBTRACTION	11	Add and subtract 2-, 3-, and 4-digit whole numbers and money amounts less than $100.00.	36
VIII.	MULTIPLICATION AND DIVISION	12	Know multiplication and division facts.	40
		13	Multiply 2- and 3-digit whole numbers and money amounts less than $10.00 by 1-digit numbers.	40
		14	Divide 2- and 3-digit whole numbers by 1-digit numbers.	40
IX.	COMPUTATION WITH FRACTIONS	15	Add and subtract fractions and mixed numbers with like denominators (without regrouping mixed numbers).	45
		16	Add fractions and mixed numbers with like denominators involving regrouping improper fractions to whole numbers or mixed numbers.	45
		17	Add and subtract fractions and mixed numbers with unlike denominators (one denominator a factor of the other).	45
		18	Find fractional parts of whole numbers.	45

PROBLEM SOLVING

	Objective		Page
X. PROCESS PROBLEM SOLVING	31	Solve process problems involving the organization of data.	51
	21	Interpret graphs, tables and charts.	51
	22	Identify the graph that best illustrates given data.	51
XI. TRANSLATION PROBLEM SOLVING	23	Identify number sentences from problems.	55
	24	Solve 1-step problems involving whole numbers and money amounts.	55
	25	Solve problems involving making change.	55
	26	Solve 1-step problems involving fractions.	55
	27	Solve 2-step problems involving whole numbers and money amounts.	55
	28	Estimate a reasonable answer to a given problem.	55
	29	Identify extraneous information in problems and solve problems with extraneous information.	55
	30	Identify needed information in problem situations.	55

MEASUREMENT AND GEOMETRY

XII. GEOMETRIC FIGURES	32	Identify geometric figures.	59
XIII. MEASUREMENT	33	Measure/determine perimeters and areas.	62
	34	Estimate lengths and areas.	62
	35	Select appropriate metric or customary units and measures.	62
XIV. TIME	36	Determine elapsed time.	67

I. COMPARING NUMBERS

Objective 1 Order whole numbers less than 100,000.

APPROPRIATE MATERIALS

Number lines
Numeral cards
Base ten blocks (Dienes or Powers of Ten)
Money
Rack-O

ENABLING SKILLS AND ACTIVITIES

Students develop a sense of number when they have many opportunities to estimate and count sets of concrete objects. A sense of the magnitude of numbers is developed by comparing sets of objects--for example, contrast a collection of 100 bottle caps with a collection of 1000. Number sense also is developed through estimation activities--have students guess how many bottle caps might fill a drinking glass, a large jar, and an aquarium.

Activities that require sorting, comparing, and ordering objects by length, area, volume, and weight all reinforce the idea of order. Such activities relate the size of objects to their corresponding numerical measure.

Place value materials provide another concrete model for comparing numbers. Ask students to use place value blocks to build a two-dimensional design or three-dimensional construction, estimate its value, and then organize and count the material to establish its value.

Activities with money reinforce place value and magnitude. Give a student some play money. Ask the student to select a certain number of pieces and make a money amount more or less than a specified amount, or between specified amounts. For example, with one-dollar bills, ten-dollar bills, twenty-dollar bills, and hundred-dollar bills available, make a set between $300 and $350, using exactly five pieces of play money. Explore the different possible solutions.

Students may also play strategy games that require them to develop efficient ways to find a "Secret Number". Have one student write a secret number on a slip of paper. Other students in turn try to guess the number.

The keeper of the secret number may only respond "It's larger," or "It's smaller," or "You've guessed it."

Counting and skip counting forward and backward aid in sequencing numbers. Have students try skip counting with interesting rules, such as count by 5 beginning at 8; or count by 20 starting at 11. Write the numbers down and look for a pattern.

The number line is a useful way of organizing numbers. Students discover patterns as they label positions on a number line.

SAMPLE LESSON Ordering Whole Numbers on a Number Line

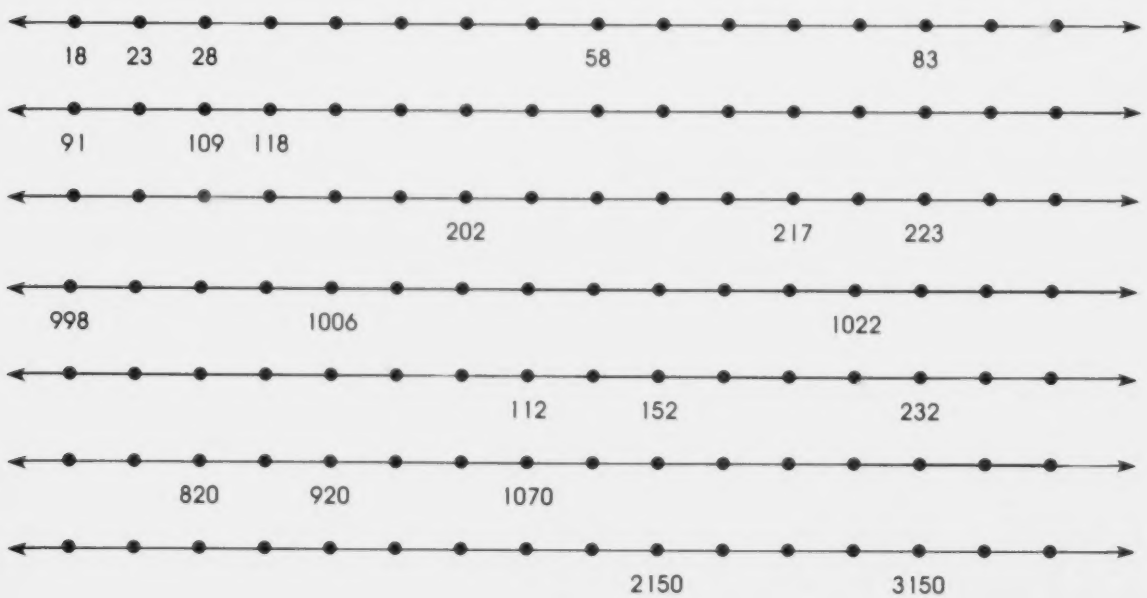
MATERIALS NEEDED

Number line sheets
Pencils

Make up a number line sheet which consists of several blank number lines. Save this as a duplicating master. Make several copies. On each sheet label some of the points on each number line. Ask the students to label the rest of the points.

Repeat the activity with other sheets. Vary the value of the interval and the size of the numbers used.

Encourage students to make up similar number line problems and exchange them with each other. Explore the patterns that emerge.



TEACHER RESOURCE MATERIAL

The Mathworks

Conceptual Understandings, continued

II. PLACE VALUE

Objective 2 Identify the value of a digit in whole numbers less than 100,000 and rewrite whole numbers using expanded notation.

Objective 3 Rename whole numbers by regrouping 1000's, 100's, 10's and 1's.

Objective 4 Round whole numbers less than 100,000 to the nearest 1000.

Objective 5 Multiply and divide multiples of 10 and 100 by 10 and 100.

APPROPRIATE MATERIALS

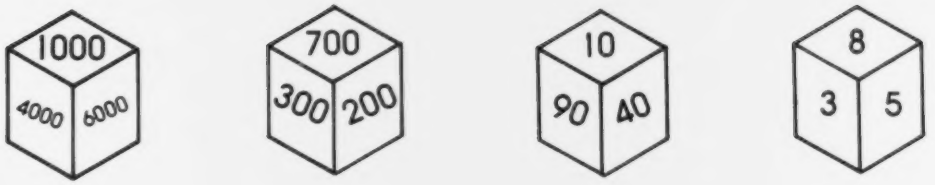
Base ten blocks (Dienes Blocks or Powers of Ten)
Beansticks and loose beans
Place value dice
Money
Place value boards
Place value charts
Place value stamps
Number lines
Grid paper

ENABLING SKILLS AND ACTIVITIES

Students develop a sense of place value and flexibility using numbers when they are provided with many opportunities to explore ways to write equivalent expressions. Students should record numbers in expanded and standard form.

Provide practice in rewriting numbers in two ways. Ask students to write a number such as 793 in the expanded form 700 + 90 + 3. Also ask them to change an expression such as 3000 + 200 + 70 + 4 to the standard form 3,274.

A quick way to generate many rewriting tasks is to give students place value dice. Place value dice can be made by finding a few cubes--use building blocks, alphabet blocks, wooden cubes made from scrap lumber, sets of large dice, or foam cubes cut from packing material. Cover each side of the dice with a piece of plain contact paper or a plain sticker. Label the sides, one number per side, to make some ones cubes (any six numbers from 0 - 9), some tens cubes (10 - 90), some hundreds cubes (100 - 900) and some thousands cubes (1000 - 9000).



Ask students to roll the set of four dice and record the result in expanded and then standard form for each roll. If students work in pairs, they can take turns rolling, recording, and checking the work.

Once students can easily record numbers in expanded form they are ready to begin renaming numbers in various regrouped forms. Familiarity with regrouped forms is a preliminary skill to the development of computational strategies. The mathematical importance of identifying and using equivalent forms to solve computational problems is established. Later, the use of equivalent forms for work with fractions, decimals, and percent will build on this foundation. Textbooks do not provide enough experience with expanded and regrouped forms. Therefore, the sample lesson for place value will focus on it.

As students develop a sense of number order and magnitude, they are prepared to "cluster" numbers based on their comparative sizes. Before students round numbers on the symbolic level, they should develop some visual models. Such models include building numbers with place value blocks and comparing several piles of place value materials for relative size. Which one of the two piles is closer in size to 600--the pile for 592 or 529?

Money also may serve as a model--is $54 closer to $50 or $60? What is the difference in dollar bills?

The number line is a pictorial model that requires the student to relate the distance between points to the relative value of a specified point. Ask students to describe the whole numbers greater than 350 that are closer to 350 than to 400 (the numbers from 351 to 374). Find the numbers on the number line. Discuss the number 375 as an example of rounding up.

Conceptual Understandings, continued

Once students develop skill rounding to tens, hundreds, and thousands, they should use rounded forms to explore alternate strategies for computation.

For example:

$$\begin{array}{r} 299 \\ +\ 436 \\ \hline \end{array}$$

may be found by rounding and adjusting

$$\begin{array}{r} 300 \\ +\ 435 \\ \hline \end{array}$$

to obtain the sum 735.

Rounded forms are clearly useful for estimation and mental computation. Once students have developed rounding skills, these skills allow the students to develop many strategies for solving computation problems. The strategies are not limited to standard procedures--students will invent interesting algorithms.

Students see patterns for multiplication and division when they multiply and divide multiples of 10 and 100 by 10 and 100. Ask students to record answers to examples and arrange them in a chart. Begin by multiplying one-digit numbers by 10, then two-digit numbers by 10, and so on. Explore multiplication and division by multiples of 10 with place value materials. Build models for each problem using place value blocks. Record the results pictorially on centimeter grid paper. Compare the results.

SAMPLE LESSON Renaming Whole Numbers by Regrouping

MATERIALS NEEDED

Place value materials (Dienes Blocks, Powers of Ten or beansticks)
Place value dice
Pencils
Record sheet (see illustration)

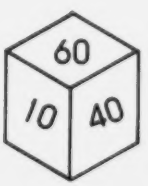
Have place value blocks available. Use the place value dice described above. Begin with three dice--hundreds, tens, and ones.

Ask the student to roll the three dice and enter the result in expanded form at the top of the record sheet. The student then writes the amount in standard form.

The student is to list different ways that the number amount may be written in regrouped form.

Ask several students to do the activity--each student rolling the dice to generate a number. Have students look at each other's results.

Repeat the activity with four dice--thousands, hundreds, tens, and ones. You may limit the number of possible solutions by using a rule, such as only regroup hundreds to tens.



Conceptual Understandings, continued

Renaming Record Sheet

Expanded Form 300 + 60 + 5	Standard Form 365
Renaming with Regrouping	
300 + 50 + 15	
200 + 160 + 5	
200 + 150 + 15	
300 + 40 + 25	

TEACHER RESOURCE MATERIAL

The Mathworks
Base Ten Mathematics

III. EQUIVALENT FRACTIONS

Objective 6 Identify equivalent fractions and mixed numbers using pictures.

Objective 7 Identify equivalent fractions and mixed numbers.

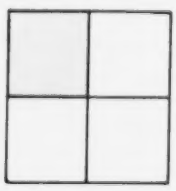
APPROPRIATE MATERIALS

Counting objects
Counting trays or egg cartons
Pattern blocks
Fraction Action game
Fit-a-Fraction Circles
Construction paper
Grid paper
Fraction strips
Fraction Bars
Proportional fraction blocks

ENABLING SKILLS AND ACTIVITIES

In the classroom, textbook activities with fractions move much too quickly to paper-and-pencil work at the symbolic level. Students attempt to learn a seemingly endless set of mysterious rules that are unconnected to the meaning of fractions. They ignore picture representations of equivalents and try to manipulate symbols according to prescribed rules. As a result, performance with fractions is poor. For instance, from the Mastery Test pilot data, students showed better performance on Objective 7 than on Objective 6. Students need a meaningful foundation for fraction concepts.

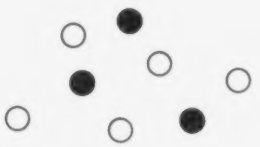
Fractions involve the idea of separating something into two or more equal parts. Each part is a unit fraction. The fraction symbol indicates the number of parts into which the unit is separated (denominator) and those parts under consideration (numerator).



$\frac{1}{4}$

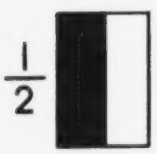
$\frac{3}{8}$

Another fraction relationship involves the fractional part of a whole number set. The fraction symbol represents one or more equal subsets of a large set of discrete objects. For example, the whole number set below has 8 objects. Three of the 8 objects are shaded--this represents 3/8.

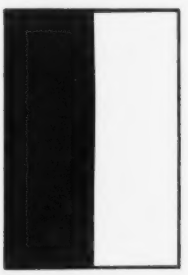


$\frac{3}{8}$

However, in either case, the size of the whole piece, or the amount of objects in a set, may differ and yet have the same fraction label. Equivalent fraction labels may represent the same ratio.



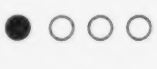
$\frac{1}{2}$



$\frac{1}{2}$

Here 1/2 is used to label one out of two parts in each case. The ratio of part to whole is the same for each picture, but the halves are not the same size (area). Similarly, half a grape is not the same size (volume) as half a basketball.

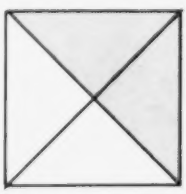
The same applies to fractions that describe parts of whole number sets. Consider 1/4 of each set as illustrated below. The ratio is the same, but the number of objects in the sets is different.



$\frac{1}{4}$

$\frac{1}{4}$

Furthermore, a fraction may be labeled in more than one way. Thus, the idea of equivalence with fractions is complex.



$\frac{1}{2}$ or $\frac{2}{4}$

$\frac{2}{6}$ or $\frac{1}{3}$

Students should have opportunities to work with manipulative materials that demonstrate the meaning of fractions. They should use a variety of materials, and record results pictorially and symbolically. This will enable them to construct visual images of fractions. Then they can recall the images and use them to connect meaning to the fraction symbols. It is the coordination among the various concrete fraction models that develops fraction concepts. The further matching of fraction models and images to the mathematical symbols will enable students to construct methods for comparing and computing with fractions.

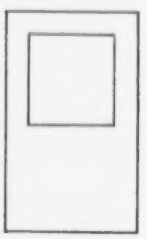
In the elementary grades we must be reasonable about the kinds of fractions that students investigate. Work with fractions whose denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, 16, and 18 is sufficient for students to develop understanding of fraction concepts.

SAMPLE LESSON Exploring Equivalence of Fractions--Halves, Fourths and Eighths

MATERIALS NEEDED

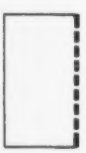
Poster or construction paper, 9" x 12" multicolor
Plain paper, 8-1/2" x 11"
Scissors
Glue
Pencils
Crayons

Draw a six-inch square on a ditto master as shown in the illustration below. Make six copies for each student.



With a paper cutter, cut the poster paper into six-inch squares (you will get two squares per sheet, plus a 3" x 9" strip that also may be used). Each student should have six squares in different colors.

Ask the students to fold one square in half. Can they find more than one way to fold the square in half?

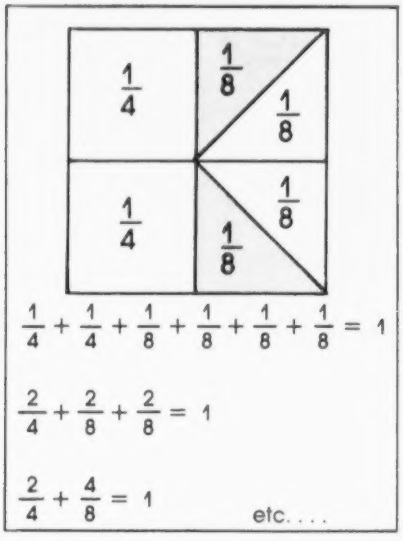


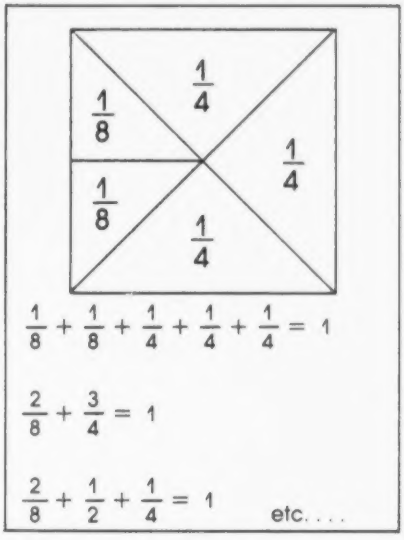
Carefully cut along the fold line. Fold another square in half and in half again. (Before it is unfolded, can students guess how many pieces they have made?) Continue to fold and cut the other squares to make halves, fourths, and eighths.

Ask each student to put all the pieces in a pile and then find a piece that is 1/2, or 1/4, or 1/8. How can the size be checked?

Ask each student to take a copy and cover the square with fraction pieces (no overlapping or white spaces showing). Glue the pieces down. Write the name of each piece on it. In the space below the square write some fraction sentences to indicate which parts make up the whole square. Look at parts that are the same color. Can different fraction sentences be written? Students will see some equivalent fractions.

Repeat the procedure with the rest of the squares.





Compare the pages made. Find the equivalents. Make a list. Discuss the results.

Repeat the procedure with other sets of fractions, such as halves, thirds, sixths, ninths, and twelfths. Put all the pages together to make a fractions book.

TEACHER RESOURCE MATERIAL

Beginning Fractions
Fractions with Pattern Blocks
Mathematics...A Way of Thinking

IV. DECIMAL CONCEPTS

Objective 8 Identify decimals (.01 to 2.99) from pictorial representations.

APPROPRIATE MATERIALS

Decimal Squares
Grid paper
Construction paper

ENABLING SKILLS AND ACTIVITIES

Decimals are an important mathematical topic. The increasing availability of calculators and computers and the increasing use of the metric system demand that decimals be given a more prominent place in the mathematics curriculum. However, as with other concepts, sufficient time must be allocated to the development of decimal concepts through the use of manipulative materials.

National- and state-level assessments of upper elementary and middle school student performance with decimals indicates that students do not understand decimals. They have difficulty identifying the place value of a digit in a two-place decimal and often ignore the decimal point and treat computation with decimals as computation with whole numbers. According to pilot data on the Mastery Test, students performed poorly on Objective 8. This indicates a lack of understanding decimals as a special case of fractions. Decimals are fractions with denominators that are multiples of ten, and decimal notation uses place value to indicate the denominators.

The study of decimals should come after students have developed basic fraction concepts and have worked with fractions whose denominators are ten. Students must have opportunities to use fraction symbols and decimal notation interchangeably as they label concrete and pictorial models. As with fractions, students need time to construct strong visual images of decimals. The images will allow them to connect meaning to the decimal notation.

SAMPLE LESSON Picturing Decimals

MATERIALS NEEDED

Construction paper (a light shade)
Glue
Scissors
Pencils
Centimeter grid paper

Ask students to use the centimeter grid paper to cut out strips that are ten squares (centimeters) long and one square (centimeter) wide. Take one strip and fold it on the first line at the left. Use pencils to shade in the region on the left side of the fold line. One square will have been shaded.

Glue the strip to the construction paper. Label it using the fraction symbol--1/10. Label it using decimal notation--0.1--and discuss the common meaning for the two symbols.

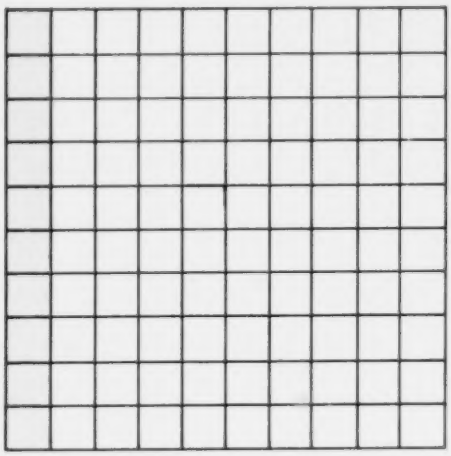
$\frac{1}{10} = 0.1$

Use another strip and fold it in half. Shade the left half. Glue the strip onto the construction paper sheet. Label it as a fraction--1/2. Ask students if it can be labeled as a fraction in another way--5/10. Label it as a decimal--0.5--and discuss the common meaning for the three symbols.

$\frac{1}{2} = \frac{5}{10} = 0.5$

Continue the procedure with more strips. Fold on other lines, shade, glue, and label.

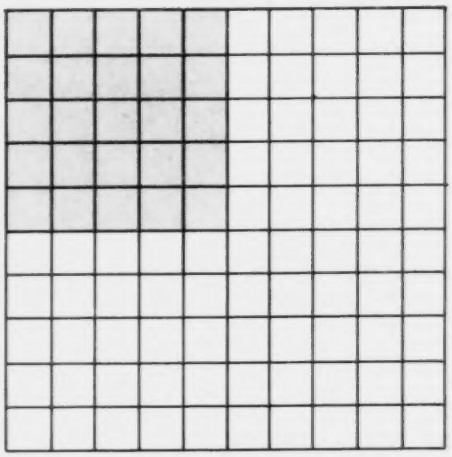
In another lesson use a similar procedure, but have students cut 10 x 10 squares from the centimeter grid paper. Fold the square along the first vertical line on the left. Shade in the region. Glue the square to construction paper (try 18" x 24" sheets) and label it as a fraction--1/10 or 10/100. Label it as a decimal--0.1 or 0.10. Discuss the results. What does each digit in the decimal represent in the picture?



$$\frac{1}{10} = \frac{10}{100} = 0.1 = 0.10$$

Continue with other squares. Let students explore various ways to shade the squares.

Conceptual Understandings, continued



$$\frac{1}{4} = \frac{25}{100} = 0.25$$

In another lesson reverse the procedure. Give students grid paper and ask them to draw and shade specific decimals that you have listed on the chalkboard.

TEACHER RESOURCE MATERIAL

The Mathworks
Mathematics: A Good Beginning
Decimal Squares
Focus on Decimals

V. PATTERNS

Objective 9 Extend patterns involving numbers and attributes.

APPROPRIATE MATERIALS

Pattern Blocks
Attribute Blocks
Color Tiles
Grid paper
Cuisenaire Rods
Geoboards
Assorted colors of 1" construction paper squares

ENABLING SKILLS AND ACTIVITIES

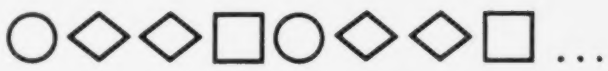
Patterning is an important area of the mathematics curriculum. The ability to classify and organize information and identify patterns helps develop good problem-solving strategies. Students must perceive similarities and differences before they can recognize and extend patterns.

The exploration of patterns should begin with the copying and extending of concrete and pictorial patterns made with materials that display different physical attributes, such as color, shape, size, or texture. Simple linear patterns may be built with Pattern Blocks. For example:

is an AB pattern with Pattern Block pieces.

Conceptual Understandings, continued

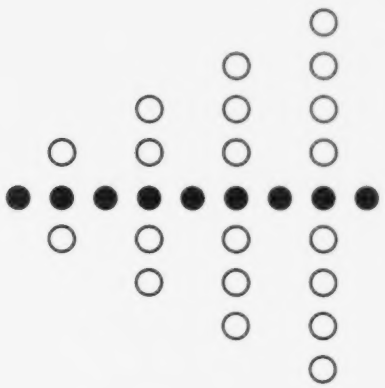
A somewhat more complex linear pattern is:



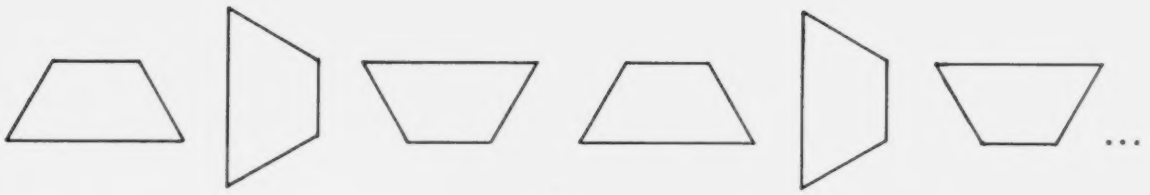
This is an ABBC pattern.

Students of all ages find patterns fascinating. Your students will find it interesting to discover how many different linear patterns are possible using Color Tiles (four colors: red, yellow, blue, green) with the rule that each color (A, B, C, D) may not be repeated consecutively in the pattern more than three times. An AABCCD pattern follows the rule; an ABBBBCD does not follow the rule.

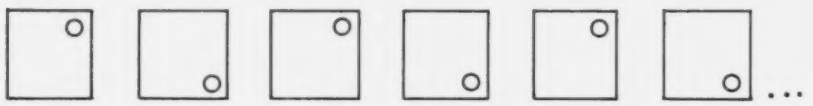
As students work with linear patterns they evolve into more complex forms, such as:



Geometrical patterns that involve rotations and flips are possible.



or



Number patterns use abstract rules for generating the pattern. Skip counting produces patterns. If you skip count by three starting at three, the rule for the pattern is "Add three," and the resulting numbers are all divisible by three. Visual patterns appear when skip counting is recorded on a hundred grid.

Simple functions also produce number patterns. For example, 2A + 1 = B has the rule, "Multiply a number by two and then add one." The result is a pattern--whether A is odd or even, B will be odd.

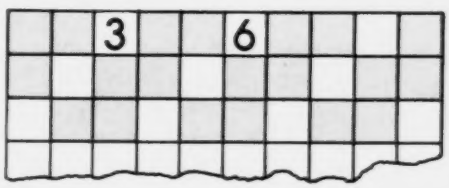
The experimentation with number patterns should be ongoing throughout the year to provide computational practice and problem-solving experiences.

SAMPLE LESSON Number Patterns on Grid Paper

MATERIALS NEEDED

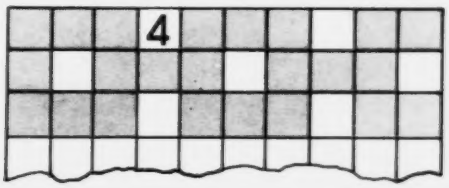
Grid paper
Pencils
Crayons

Use a grid that is ten squares wide and eight to ten squares long. Use crayon to color in two consecutive squares; skip a square and repeat the color pattern, continuing it to the beginning of each line (this is an AAB pattern with the "B" boxes left blank).



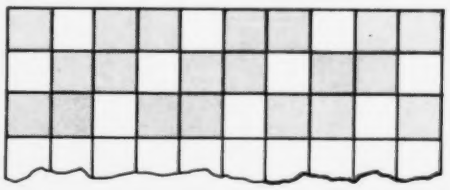
With pencil, label the first blank square 3. The next blank square will be labeled 6. Continue to label the blank squares in the first three rows. Is there a pattern? What is the rule? Try to label the blank squares along a diagonal. What is the rule?

Using another sheet of grid paper, color in a different pattern, AAAB perhaps. Label the first blank square 4. Finish labeling the blank squares by working across, on the diagonal or moving vertically.

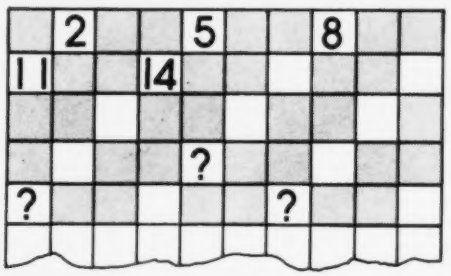


Use other grid sheets to try more patterns of this kind.

Then try a more complex pattern. Color the grid paper as follows: color the first box, skip the second box, then start an AAB pattern in the third box, coloring in the "A" boxes.



Label the blank squares. What is the rule for this pattern? What is the rule if you move diagonally? What is the rule if you move vertically? Try filling in any square at random without counting.



Try other variations of these patterns on grid paper.

TEACHER RESOURCE MATERIALS

Let's Pattern Block It
Problem-Mathics
Pattern Block Activities
Tessellations: The Geometry of Patterns
The Mathworks

VI. ESTIMATION

Objective 10 Identify an appropriate procedure for making estimates for whole number computations.

Objective 19 Estimate sums and differences of whole numbers and money amounts.

Objective 20 Estimate products and quotients of whole numbers and money amounts (1-digit factor and 1-digit whole number divisor).

APPROPRIATE MATERIALS

Money
Number lines
Newspapers, magazines, and store fliers
Calculators

ENABLING SKILLS AND ACTIVITIES

The ability to estimate the results of computations is an essential skill. With the increasing use of calculators and computers, estimation is more commonly used than paper-and-pencil computations to judge the reasonableness of an answer displayed on a machine.

Students should develop estimation skills as they develop computational strategies. The teaching of estimation should not be limited to rounding, but should involve the exploration of many strategies including compensation, clustering, finding compatible numbers, front-ending, and recognizing computational patterns with special numbers. Have students discuss their different approaches to the same situation.

Estimation activities can be integrated into other curriculum topics, such as measurement. The relationship between problem solving and estimation should be emphasized; looking back at a solution includes judging the reasonableness of the answer obtained.

Some prerequisite skills for successful computational estimation include the ability to do 1- and 2-digit computations, to work with rounded numbers, and to compare and order numbers. Students must have an understanding of place value concepts and the nature of the decimal number system and its basic operations.

In addition to developing skills in rounding, place value, and operating with rounded numbers, it is essential that students develop a sense that multiplication and division in general have much greater effects than do addition or subtraction on the order of magnitude of the result. Students should practice operating with tens--adding and subtracting tens and multiples of ten. Multiplying and dividing by multiples of ten should be carefully explored to further develop an understanding of the size of a number.

Computational estimation activities should be accompanied by work with pictures or models such as number lines or place value materials. Students also need to estimate capacity, length, area, and volume with real materials to enhance their sense of number. Such activities lead them to discover that estimates are easier to use. It is important for students to develop a tolerance for guessing and accept that more than one answer may be reasonable.

Estimation problems may be presented in a variety of formats. Some problems require the establishment of a reference number; for example, "Will $30 be enough to pay the grocery bill?" Some require a decision about the reasonableness of an answer; for example, "The calculator shows 324 multiplied by 8 as 6592. Is that right?" Some are open-ended; for example, "About how much is 3478 x 42?"

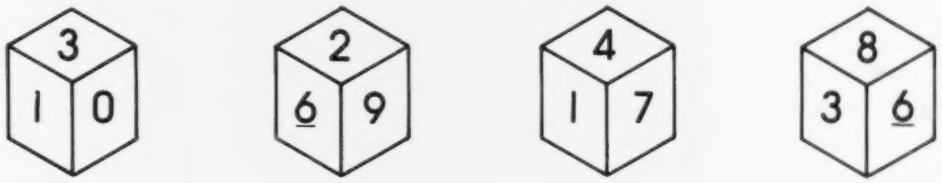
SAMPLE LESSON The Special Number Range Game- An Estimation Activity

MATERIALS NEEDED

A set of four numeral dice
Paper and pencils
Calculators

The game is played by two or more players or teams. You may introduce it to the whole class to clarify the rules.

Make two red numeral dice and two blue numeral dice. Randomly label each face with any one digit 0 - 9.



red red blue blue

To begin the game, roll the four dice. Arrange the two red ones to form the smallest possible number. Arrange the two blue ones to form the largest possible number. The two numbers will serve as the range. Record the two numbers.

23 84

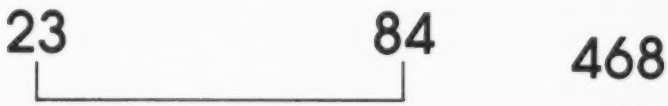
Next, roll three of the dice, one blue and two red. This roll will determine the Special Number. Use the number on the blue die to make the hundreds position, and the numbers on the red dice for tens and ones. Write down the Special Number.

blue

red

red

468



468

The players alternate asking and answering questions about the Special Number and the range. Sample questions include:

What number can you subtract from 468 to get inside the range?

What number can you divide 468 by to get a number inside the range?

Check the estimate with a calculator.

Players roll the dice and repeat the procedure for a total of five rounds. The player who gets the most answers inside the range wins.

The activity can be made easier by rolling only one or two dice to make the Special Number. The activity can be made more challenging by rolling four dice to make the Special Number.

TEACHER RESOURCE MATERIALS

Developing Computational Skills: The 1978 NCTM Handbook
Estimation and Mental Computation: The 1986 NCTM Handbook
Krypto
Developing Skills in Estimation
Guess: Guide to Using Estimation Skills and Strategies
Calculator Explorations and Problems

VII. ADDITION AND SUBTRACTION

Objective 11 Add and subtract 2-, 3-, and 4 digit whole numbers and money amounts less than $100.00.

APPROPRIATE MATERIALS

Place value material (Dienes Blocks and Powers of Ten)
Place value boards
Place value cards
Money

ENABLING SKILLS AND ACTIVITIES

The written procedures for addition and subtraction involve the manipulation of symbols. Traditionally we have relied on the student's ability to memorize algorithms that direct the steps involved in computation. However, many of the procedures quickly become separated from the reality of the numbers being operated on. The procedure of borrowing and carrying, for example, is so divorced from the quantities involved that it makes no difference whether one is dealing with units, tens or hundreds, or even decimals--a number to the left is crossed off, a value one less than the number is written above it, and a little numeral one is placed in front of the number on the right.

$$\begin{array}{r} {}^{1}\not{1}\,\not{2}\,1 \\ -\;\;9\,7 \\ \hline \end{array} \qquad \begin{array}{r} {}^{2}\not{3}\,09 \\ -\;2\,8 \\ \hline \end{array}$$

When students apply such procedures prematurely, they arrive at an unquestioned "answer". Errors that creep into the algorithm are undetected and "senseless" mistakes, such as subtracting and obtaining a value that is larger than the minuend, are made.

Research findings indicate that students need to spend more time dealing with quantities through the manipulation of concrete materials that preserve the meaning of the numbers and operations. Research also has shown that students can construct various methods for dealing with computation at the symbolic level if they are allowed to monitor their work through the use of concrete models and pictures. The development of nonstandard computational strategies provides more flexibility, not only with paper-and-pencil calculations, but also with estimation and mental computations.

Arithmetic should not be seen as a collection of meaningless rules, but as an opportunity to think. Inventing computational strategies and alternate algorithms is problem solving.

SAMPLE LESSON Computation as Problem Solving

MATERIALS NEEDED

Paper and pencil
Place value material

Have Dienes Blocks or Powers of Ten material available for those students who wish to use them.

Begin with the whole class. Ask students to write down all the different ways to find the sum of 59 and 72.

In the beginning many students will indicate the use of the standard algorithm.

$$\begin{array}{r} {}^{1}\,\\ 59 \\ +72 \\ \hline 131 \end{array}$$

Ask students if they can think of other ways to work with the numbers. Discuss each suggestion as it is made. Students will begin to put variations together and may make suggestions such as:

$$\begin{array}{r} 59 \\ +72 \end{array} \Rightarrow \begin{array}{r} 60 \\ +72 \\ \hline 132 \\ -\ 1 \\ \hline 131 \end{array}$$

$$59 + 72 \Rightarrow 60 + 71 = 131$$

$$59 + 72 \Rightarrow 50 + 70 + 9 + 2 \Rightarrow$$
$$120 + 11 = 131$$

Repeat the procedure with other addition problems. Then allow the students to work in pairs or small groups. Give them similar problems to do together. Have groups compare their strategies.

Repeat the procedure with larger numbers. Look for patterns. Is it easier to add 18 by adding 20 and then subtracting 2?

$$77 + 18 \Rightarrow 77 + 20 - 2 \Rightarrow$$
$$97 - 2 = 95$$

Then ask students to investigate different ways to subtract. A few strategies they might suggest include:

$$\begin{array}{r} 37 \\ -19 \\ \hline \end{array}$$

$$37 - 19 \Rightarrow 38 - 20 = 18$$
$$37 - 19 \Rightarrow 37 - 20 + 1 = 17 + 1 = 18$$
$$37 - 19 \Rightarrow 30 - 20 + 8 = 10 + 8 = 18$$

Continue the activity as an ongoing way to practice number work and improve computational skill. Ask students to try some mental computations. When paper and pencil is not used, do they find nonstandard strategies more useful?

TEACHER RESOURCE MATERIALS

Learning from Children
Developing Computational Skills: The 1978 NCTM Handbook
Journal for Research in Mathematics Education, January 1985

VIII. MULTIPLICATION AND DIVISION

Objective 12 Know multiplication and division facts.

Objective 13 Multiply 2- and 3-digit whole numbers and money amounts less than $10.00 by 1-digit numbers.

Objective 14 Divide 2- and 3-digit whole numbers by 1-digit numbers.

APPROPRIATE MATERIALS

Place value materials (Dienes Blocks or Powers of Ten)
Counters
Grid paper
Money

ENABLING SKILLS AND ACTIVITIES

If students are not able to state all the multiplication and division facts, the use of number patterns and the multiplication/division relationship can help to organize the facts into those they know, and those facts that they may acquire by linking to known number facts. Memorization of facts for long-term retention is best done by exploiting connections to remembered information. Students will be reassured to see that they have mastered many facts and can reconstruct facts that are not immediately recalled.

Multiplication on the concrete counting level is demonstrated as the combining of several subsets, each subset containing the same number of objects. This is the repeated addition model (4 x 3 is 3 + 3 + 3 + 3). Division is demonstrated as the reverse operation--the repeated subtraction of subsets that are the same size.

Students often have rather undeveloped ways of "doing" division. If you give a student a pile of objects and ask him to "divide them up among five students," you may observe the student deal the objects out one at a time. Sometimes, a more sophisticated counting strategy--dealing the objects out in groups of two, three, five, or ten--is used. Rarely does a student count all the objects and then apply the appropriate division by five fact. Students do not readily connect the strategies for doing division with counting objects to the procedures for doing division with paper and pencil. These connections can be encouraged by using concrete and pictorial models as the symbolic work with division is developed.

The mastery of basic multiplication and division concepts is simplified when students understand that the relationship among two factors and their product can be used to find the basic facts.

Counters arranged to display the repeated addition model may be rearranged in an array.

$$2\overline{)\begin{matrix}\bigcirc & \bigcirc & \bigcirc & \bigcirc \\ \bigcirc & \bigcirc & \bigcirc & \bigcirc\end{matrix}}\ 4 \qquad 2\overline{)\,8}\ 4 \qquad 2 \times 4 = 8$$

$$f\overline{)\,p}\ f \qquad f \times f = p$$

When the array is labeled, a clear relationship between multiplication and division is established. The two factors and product displayed are also consistent with the standard symbol used for division.

It is helpful to use the factor-factor-product terminology rather than to introduce more terms, like dividend and quotient. Division is the process of finding a missing factor. When students see the reversibility of the procedure they find that there are fewer facts to remember. The definition of division as the search for a missing factor is also consistent with its algebraic treatment.

Once students have mastered the basic facts and concepts, the development of strategies for multiplication and division is appropriate. Students should have some opportunity to work computation problems in expanded form so that the implications of place value can be explored.

There are several different algorithms for doing multiplication and division with paper and pencil. All of them must account for the value of the digits in the factors and product. Counters and place value blocks may be used to illustrate the role place value plays in any algorithm. Time must be spent working at this level before moving to work totally at the symbolic level. With an understanding of the role of place value in multiplication and division, students will be able to make good computational estimates and also will develop strategies for mental computation.

SAMPLE LESSON Exploring the Role of Place Value in Multiplication and Division Through Expanded Forms

MATERIALS NEEDED

Place value blocks
Grid paper
Pencils

Have place value blocks available so that students may explore various ways to decompose and rearrange the numbers.

Begin with the whole class. Ask them to write down all the different ways to find the product of 67 times 5.

At first many students will indicate the use of the standard algorithm.

$$\begin{array}{r} {\scriptstyle 3} \\ 67 \\ \times\ 5 \\ \hline 335 \end{array}$$

Ask students if they can think of other ways to work with the numbers. Discuss each suggestion as it is made. Students will begin to invent strategies and may make suggestions such as:

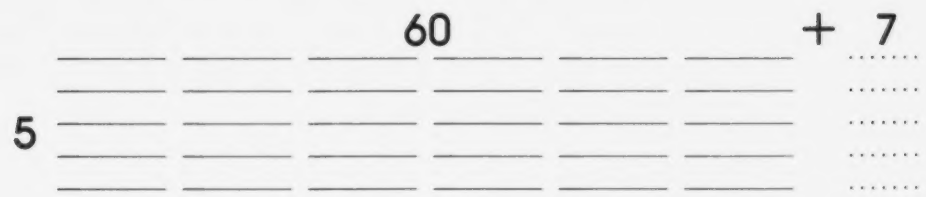
$$67 \times 5 \Rightarrow 60 \times 5 + 7 \times 5 \Rightarrow$$
$$300 + 35 = 335$$

$$\begin{array}{r} 67 \\ \times\ 5 \\ \hline 35 \\ 300 \\ \hline 335 \end{array}$$

$$67 \times 5 = (60 + 7) \times 5$$

Students may wish to build concrete models to see the effect of place value on the partial products.

_____ represents a 10-rod.



	60	+ 7
5	300	35

Repeat the procedure with other multiplication problems. Allow students to work in pairs or groups. Give each group the same problem. Have groups compare their models and their written strategies.

Repeat the procedure with larger numbers. Look for patterns.

In another lesson explore division. Students may come up with strategies such as:

$$3\overline{)45} \Rightarrow 3\overline{)\overset{10+5=15}{30+15}}$$

$$4\overline{)128} \Rightarrow 4\overline{)\overset{30+2=32}{120+8}}$$

$$9\overline{)108} \Rightarrow \qquad \overset{10 + 2 = 12}{9\overline{)90 + 18}}$$

Continue the activity as an ongoing way to practice number work and improve computational skills. Ask students to try some mental computations. When paper and pencil are not used, do they find some of their nonstandard strategies useful?

TEACHER RESOURCE MATERIALS

The Mathworks
The I Hate Mathematics Book
Math for Smarty Pants
Base Ten Mathematics
Those Amazing Tables

IX. COMPUTATION WITH FRACTIONS

Objective 15 Add and subtract fractions and mixed numbers with like denominators (without regrouping mixed numbers).

Objective 16 Add fractions and mixed numbers with like denominators involving regrouping improper fractions to whole numbers or mixed numbers.

Objective 17 Add and subtract fractions and mixed numbers with unlike denominators (one denominator a factor of the other).

Objective 18 Find fractional parts of whole numbers.

APPROPRIATE MATERIALS

Fraction strips
Fraction Circles
Fraction Bars
Proportional fraction blocks
Pattern Blocks
Cuisenaire Metric Blocks
Fraction books (made by students)

ENABLING SKILLS AND ACTIVITIES

It may appear simpler, both to teacher and student, to just put in place a few computational rules for dealing with fractions. We ask students to memorize rules because we have given up the idea that students can understand fractions, or that there is any advantage to understanding them. However, an understanding of concepts for fraction computations, in fact, will make the process simpler. Estimation and mental computation with fractions are possible only if fraction concepts are grasped. Even paper-and-pencil computation is easier if concepts are understood well enough to allow for the use of simplified strategies. For example, if only the rule for multiplying two fractions is used, students do the computation as follows:

$$\frac{3}{8} \times \frac{5}{5} = \frac{15}{40} \div \frac{5}{5} = \frac{3}{8}$$

However, with an understanding that $\frac{5}{5} = 1$

the answer more readily follows.

$$\frac{3}{8} \times \frac{5}{5} \Rightarrow \frac{3}{8} \times 1 = \frac{3}{8}$$

Multiplication is a natural place to begin computational work with fractions. The models involved are easy to visualize. To find a fractional part of a whole number set, students need to understand the part-to-whole relationship and that the parts are all the same size. On the other hand, addition and subtraction of fractions not only require the part-to-whole relationship but also the idea of equivalence.

SAMPLE LESSON Developing Computational Strategies for Fractions

MATERIALS NEEDED

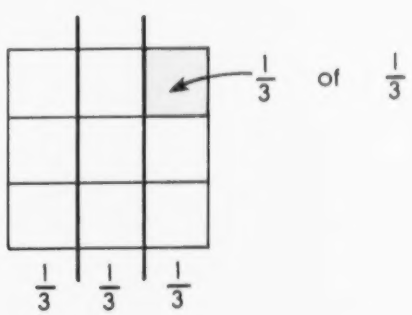
Poster or construction paper
The student-made fraction books from the Sample Lesson--
Exploring Equivalence of Fractions (Objectives 6 and 7)
Scissors
Glue
Pencils

Let's begin with multiplication and follow a sequence with several models. To find 1/3 of 45, three equal subsets must be built from 45 counters. The quantity in one subset is the answer--1/3 of 45 is 15. The sentence is written as 1/3 x 45 = 15.

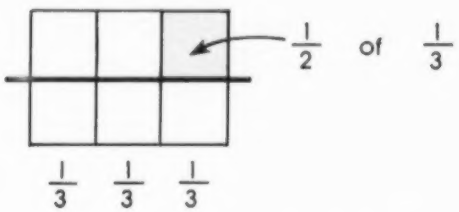
To find 2/3 of 45, the amount is again allocated to three equal subsets. The quantity in two of the subsets is the answer. Therefore, 2/3 of 45 is 30. Three-thirds would be all three subsets, or 45.

Students should practice partitioning sets and labeling the results. They will begin to connect the multiplication/division facts with the operation and will not need to physically construct the set and subsets.

Before going on with other operations with fractions, students must develop the idea of equivalence. The sample lesson for Objectives 6 and 7 used student-made fraction books as one model for developing equivalence. Often, as students work with the books, they spontaneously volunteer another way to label a section, such as 1/9.



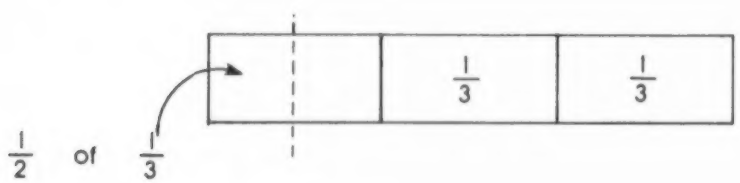
This is a good model to use for extending the operation to the multiplication of two fractions. Students should make more pages in their fraction books. Have them label the sections to name what part one fraction is of another--1/2 of 1/3 is 1/6.



Now there are 6 parts. Each part is 1/6.

This is written as 1/2 x 1/3 = 1/6.

Folding fraction strips is another model. Fold the unit length in thirds. Fold one-third in half.



Students should continue this process and then look for patterns. For example, make a list like this:

1/2 x 1 = ____

1/2 x 1/2 = ____

1/2 x 1/3 = ____

1/2 x 1/4 = ____

1/2 x 1/5 = ____

1/2 x 1/6 = ____

What is the "rule"?

What is 1/2 x 1/12?

What is 1/2 of 2/12; or 1/2 of 4/12; and so on? Explore the patterns visually and symbolically.

Limit the denominators to 2, 3, 4, 5, 6, 8, 9, 10, 12, 16, and 18. These are realistic in terms of practical use and the set of them is small enough to allow students to explore them all at concrete and pictorial levels.

With the same limited set of eleven denominators, begin to explore addition and subtraction. Students who have spent time writing fraction sentences for the pages in their fraction books will already have determined a way to add with the model. Have them review their sentences. Write more sentences that describe parts of a fraction picture. For example, 1/3 + 1/3 = 2/3; or 1/4 + 2/4 = 3/4. During addition and subtraction with like denominators students will count the parts. Ask them to write many sentences with like denominators.

The work with like denominators will naturally evolve into writing sentences with unlike denominators. If students have had sufficient time with the fraction books, they will not suggest that 1/2 + 1/4 is 2/6. The visual model that they have developed will help them think through the situation. They will see that 1/2 + 1/4 covers 3/4. They will also know that 1/2 is the same as 2/4. The problem can be rewritten as 2/4 + 1/4. There is no need to work with least common multiples and greatest common factors. Nor do we need to find the common denominator through the application of rules at the symbolic level. Allow the students to think their way through using fraction concepts.

Once students have automatically developed their own strategies, extend the work with fractions to include mixed numbers. With models, as well as the distributive property already used in work with whole numbers, students will extend their strategies to organize problems such as

1/2 of 1-1/2 as 1/2 of 1 and 1/2 of 1/2

1/2 x 1-1/2 = 1/2 x 1 + 1/2 x 1/2 =

1/2 + 1/4 = 3/4

TEACHER RESOURCE MATERIALS

Beginning Fractions
Fractions with Pattern Blocks
Solid Sense in Mathematics 4-6

X. PROCESS PROBLEM SOLVING

Objective 31 Solve process problems involving the organization of data.

Objective 21 Interpret graphs, tables and charts.

Objective 22 Identify the graph that best illustrates given data.

APPROPRIATE MATERIALS

Attribute blocks
Tangrams
Geoboards
Pattern blocks
Place value material
Money
Grid paper
Calculators
Computers

ENABLING SKILLS AND ACTIVITIES

There are two areas of problem solving that are developed in a curriculum that focuses on mathematical thinking--process problems and translation problems. Translation problems (also called word or story problems) require the problem solver to operate at a symbolic level. Words are translated into mathematical symbols, mathematical operations are done with the symbols, and the resulting mathematical sentence is translated back to words. The problem solver may not have seen a particular word problem before (the combination of words is new) but the problem may be translated into a mathematical statement which uses familiar symbols, operations, and concepts.

Although words and symbols are involved in process problems, the context is different. Process problem solving involves working with novel situations. The context of the problem may require spatial skills. For example, the problem may entail finding all the different rectangles that can be constructed on a geoboard. The problem solver must find a systematic way to find all possible solutions.

A variety of strategies may be utilized during the process of solving the problem. Guess-test-revise is a simple approach. Other strategies include building a model, drawing a picture, or acting out the problem. Information from the problem may need to be organized. A list, table or graph may be employed. Once the information is organized, the problem solver may search for patterns or reorganize the problem into smaller, simpler parts. The problem solver may use the components to work backward or to analyze the pieces logically.

For students to develop these strategies, the curriculum must emphasize classifying, patterning, graphing, estimating, and making predictions. Students must have many opportunities to analyze and structure situations.

Students must be:

- faced with problems in which the approach is not apparent and encouraged to generate and test many alternative approaches;
- allowed sufficient time for discussion, practice, and reflection on problems and problem-solving strategies;
- encouraged to solve different problems with the same strategy and to apply different strategies to the same problem;
- provided many experiences solving problems in small groups; and
- involved in the collection and organization of data.

SAMPLE LESSON How Many Ways Can You Make $2.50?

MATERIALS NEEDED

Paper and pencil
Record sheet
Play money

Ask students to work in pairs or small groups. Give each student a blank piece of paper. Have available sets of play money--pennies, nickels, dimes, quarters, half-dollars, and dollars. Ask the students to find and list all the different ways they can make $0.42 using pennies, nickels, dimes, and quarters.

Observe how the students approach the problem. Some students may need to rearrange the play money to help them find all the combinations. They may begin to randomly find ways to make the amount. Other students may organize the task and use just pennies, then pennies and nickels, and so on. Some students may organize the information into a table.

Have groups share their strategies. Ask the students to repeat the procedure to find all the coin combinations that make $0.83. Discuss the results. Give each group one record sheet and ask them to find all the combinations for $2.50.

As an extension to the activity, ask students to find the different values that can be made with one coin, two coins, three coins, and so on.

Record Sheet for Money Combinations $2.50

Pennies	Nickels	Dimes	Quarters	Half-Dollars	Dollars
250					
245	1				
240	2				
240		1			

TEACHER RESOURCE MATERIALS

Attribute Acrobatics
SPACES: Solving Problems of Access to Careers in Engineering and Science
Visual Thinking
Spatial Problem Solving with Paper Folding and Cutting
What's My Logic?
Make It Simpler
Building Thinking Skills
Math for Girls and Other Problem Solvers
Problem Solving in School Mathematics: The 1980 NCTM Yearbook

XI. TRANSLATION PROBLEM SOLVING

Objective 23 Identify number sentences from problems.

Objective 24 Solve 1-step problems involving whole numbers and money amounts.

Objective 25 Solve problems involving making change.

Objective 26 Solve 1-step problems involving fractions.

Objective 27 Solve 2-step problems involving whole numbers and money amounts.

Objective 28 Estimate a reasonable answer to a given problem.

Objective 29 Identify extraneous information in problems and solve problems with extraneous information.

Objective 30 Identify needed information in problem situations.

APPROPRIATE MATERIALS

Calculators
Computers

ENABLING SKILLS AND ACTIVITIES

Translation problems require the problem solver to change a story problem from words to mathematical symbols. The symbols are manipulated through the use of the appropriate mathematical operations. The resulting mathematical statement is translated back to words and the answer is checked for reasonableness in terms of numerical magnitude and its fit within the context of the problem. Solving translation problems requires the student to operate at an abstract level.

To successfully solve translation problems at the upper elementary level, students must have good reading comprehension and good skills applying mathematical operations (addition, subtraction, multiplication, and division) with various kinds of numbers (whole numbers, fractions, decimals, and integers). Students must understand that number sentences provide specific information, that they are general statements that may be used to describe similar situations, and that the components of word problems provide pieces of information that help develop the number sentences.

The following sequence of experiences is helpful in developing translation problem skills:

Match a number sentence to a picture or diagram, and match a diagram to the appropriate number sentence.

Draw a diagram to illustrate a word problem.

Write questions that may be answered from given pieces of information.

Match number sentences to word problems.

Write number sentences to fit word problems.

Some classroom teaching strategies for developing translation problem-solving skills include activities in which students practice reading skills such as selecting main ideas. Encourage students to put problems into their own words. Have students note the important information given in a problem, and have them identify useless or confusing information. Ask students to determine the question to be answered, select specific information necessary for the solution, and choose the appropriate operation(s).

Provide opportunities for students to work in pairs or small groups. Have them estimate answers and discuss strategies for making the estimates.

After completing the problem, have students look to see if they might have solved it differently. Ask students to test the reasonableness of their answers.

SAMPLE LESSON Creating and Publishing Story Problems

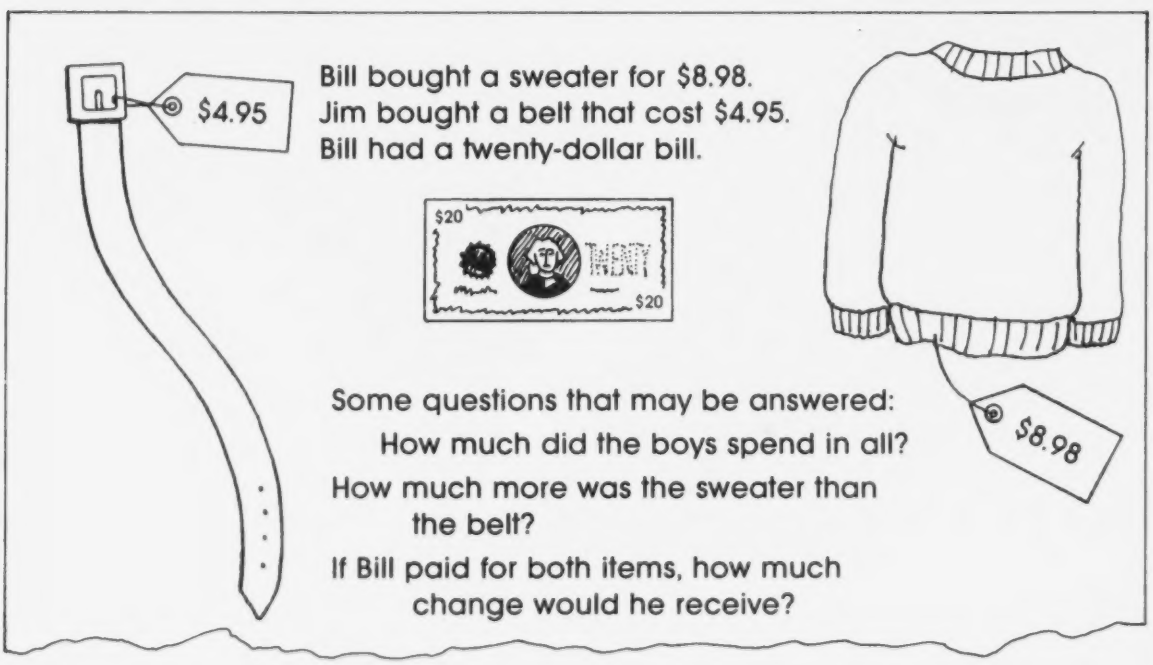
MATERIALS NEEDED

Paper and pencil
Crayons, magazines, and/or computer graphics

When students write their own story problems, they develop the organizational skills needed to solve problems. They also analyze information and generalize ideas.

You can organize story problem writing as a class project. Have students work in small groups. Their task is to write, illustrate and publish story problem books.

Each student should create some pieces of information. They are to exchange the information scenarios with other students in the group. Students are to write questions that can be answered from the information, illustrate the situations, and compile them into a book.


$4.95
Bill bought a sweater for $8.98.
Jim bought a belt that cost $4.95.
Bill had a twenty-dollar bill.
$20
$20
$8.98
Some questions that may be answered:
How much did the boys spend in all?
How much more was the sweater than the belt?
If Bill paid for both items, how much change would he receive?

The stories may be illustrated by any combination of student drawings, pictures cut from old magazines, or computer graphics generated with software such as Newsroom or Printshop. The group should produce an answer page.

Place the story problem books at an activity center and have students read them and do each others' problems. Share the books with other classrooms. Have each student select a favorite story, and collect and reproduce them as a story problem newsletter that is distributed to parents.

TEACHER RESOURCE MATERIALS

Mind Benders
Wollygoggles and Other Creatures
Arithmetic Teacher, February, 1982 (Focus Issue on Teaching Problem Solving)
Problem Solving In School Mathematics: The 1980 NCTM Yearbook

XII. GEOMETRIC FIGURES

Objective 32 Identify geometric figures.

APPROPRIATE MATERIALS

- Geoboards
- Pattern Blocks
- Tangrams
- Pentominoes
- Mira
- Wooden or plastic geometric solids
- Cuisenaire Metric Blocks
- Cuisenaire Rods
- Cuisenaire Polydrons
- Centimeter cubes
- Orbit kit
- Protractors
- Compasses
- Grid paper

ENABLING SKILLS AND ACTIVITIES

Geometry topics provide a rich and enjoyable strand in the mathematics curriculum. The development of spatial skills is an important part of an elementary program that focuses on thinking skills.

The study of geometry should not be restricted to the memorization of a vocabulary. Students must work with manipulative materials in order to explore spatial relationships. Students should use materials such as tangrams, Pattern Blocks, and geoboards to construct two-dimensional shapes. Manipulatives such as centimeter cubes, metric blocks, rods, polydrons, and the Orbit kit may be used to explore three-dimensional solids. At the pictorial level students should use protractors, compasses, straightedges and Miras to reproduce shapes and create representations of the solids.

Geometrical ideas such as shape, size, length, area, volume, congruence, similarity, and symmetry can all be discovered during exploration activities with manipulatives. Spatial relationships also can be investigated by considering the transformation of shape pieces as they are rotated or flipped.

Shapes and solids can be compared and classified by size, number of sides, number of angles, types of angles, or number of faces. Vocabulary should be built as a natural extension of classification activities.

SAMPLE LESSON Classifying Geometric Shapes

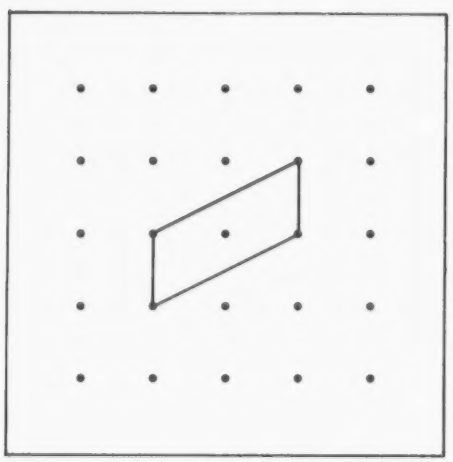
MATERIALS NEEDED

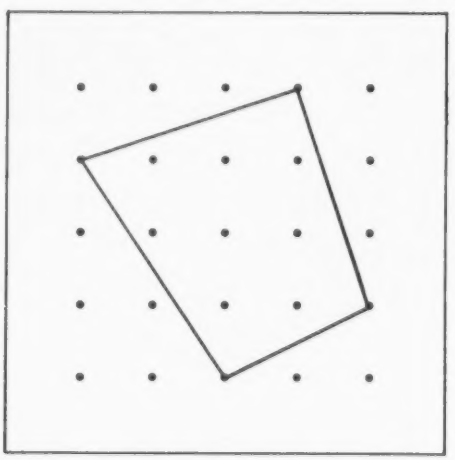
Geoboards and geobands
Geodot paper
Pencils

Have students work in small groups. Give each student a geoboard, a geoband, and several sheets of geodot paper.

Tell the students to construct a shape on the geoboard with one band. The only requirement is that the band must touch exactly four pegs. Then have the students use pencil to reproduce the shape on one section of the dot paper.

Ask students to clear the geoboard and then build another, different shape on the geoboard that follows the same rule--one band that touches exactly four pegs. Students should reproduce the shape on another section of the geodot paper.





It is possible to produce many shapes while working with the same rule. Have students repeat the procedure to make 8 - 12 designs each.

Then ask them to cut the geodot designs apart and place them in the center of the group's workspace. Each small group of students should then work together to sort the designs. The students should decide how to sort them.

Once the designs have been sorted, ask students to try and find the special names for the designs in each category. Students should use textbooks and other reference materials such as dictionaries and mathematics resource books.

The students should compare the results of each small group. Did they find names like quadrilateral, trapezoid, parallelogram, rhombus, and so on? Which names can be used to describe more than one group of designs?

Repeat the process to investigate the classification of other shapes. Use another rule, such as one band that touches exactly three pegs, to explore the different kinds of triangles.

TEACHER RESOURCE MATERIALS

Mira Math for Elementary School
The Mirror Puzzle Book
Visual Thinking Cards
Spatial Problem Solving with Cuisenaire Rods
Spatial Problem Solving with Paper Folding and Cutting
Let's Pattern Block It

XIII. MEASUREMENT

Objective 33 Measure/determine perimeters and areas.

Objective 34 Estimate lengths and areas.

Objective 35 Select appropriate metric or customary units and measures.

APPROPRIATE MATERIALS

Color Tiles
Geoboards
Geodot paper
Centimeter cubes and base ten rods (Powers of Ten)
Tangrams
Pattern Blocks
Construction paper squares
Grid paper (inch and centimeter)
Transparent grids
Ceramic tiles
Cuisenaire Metric Blocks
Rulers and meter sticks

ENABLING SKILLS AND ACTIVITIES

Students should continue to explore the measurement of length and area with standard and nonstandard units. Often, real life situations which require students to estimate area or length do not require them to use standard units. They may need to estimate how many tiles will cover a floor, how many pieces of border trim will outline a bulletin board or how many rocks will edge a garden. Estimation practice with nonstandard units helps develop spatial skills.

Use objects such as paper clips, toothpicks, and straws for measuring length. Estimate and measure with nonstandard units such as handspans, footspans, and strides. Use large dry lima beans, ceramic tiles, and construction paper squares for measuring area.

Estimation practice with standard units helps students develop a frame of reference for the sizes of units of measure. Don't just use a ruler or meter stick; other materials that yield standard measures include Cuisenaire Powers of Ten rods, Cuisenaire Metric Blocks, and Color Tiles. The advantage with

these materials is that they are physically countable; this reinforces the idea that measurement is done by repeatedly using a uniform unit. Be sure to measure the same lengths or areas with different nonstandard and standard units so that students can compare units.

When students are provided with many opportunities to estimate and measure, they understand the physical attributes of length, area, and perimeter. They also construct a mental image of the unit of measure that helps them decide whether it is an appropriate unit; for example, should we use a centimeter or a meter as the unit when we measure the length of the gymnasium?

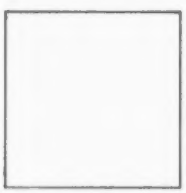
As students deal with area, provide them with opportunities to organize and compare direct measures of length and area. Do not tell students to use a certain formula--allow them to explore the relationships between lengths of sides and the area of shapes. They will discover some patterns and generalize the formula.

SAMPLE LESSON Organizing Information About Sides and Area

MATERIALS NEEDED

Color Tiles
Pencils
One-inch grid paper
A record sheet (see illustration)

Each student will need 30 Color Tiles, several sheets of grid paper, a pencil, and a copy of the record sheet (if you do not have Color Tiles, you may use ceramic tiles or construction paper squares). Tell the students to make the smallest possible square. Ask them to draw it on the grid paper. Discuss the characteristics of the square--each side spans the distance between adjacent lines on the grid and there are no lines in between. We will call the length one unit; the square has a length of one and a width of one. We will call the area of the square one square unit. Label the drawing and write the measures on the record sheet.



Record Sheet		
Length	Width	Area — square units
1	1	1

Next ask the students to build a rectangle with two tiles. Ask students to describe its sides (length and width) and area. Draw the rectangle on grid paper and label its sides. Record the measures on the record sheet.

Record Sheet		
Length	Width	Area — square units
1	1	1
2	1	2

Continue the procedure of making rectangles with 3, then 4 tiles and so on to 30. With certain numbers of tiles more than one rectangle is possible. Record all the variations that are found; some will be the special case rectangle--the square. Draw and label the rectangles and complete the record sheet.

Discuss the results. Is there a pattern? Can anyone guess what the sides might measure for a rectangle made with 31 tiles, or 32 tiles? What would be the area of a rectangle built with a length of 8 tiles and a height of 5 tiles? Make more predictions. What is the rule?

Record Sheet		
Length	Width	Area - square units
1	1	1
2	1	2
3	1	3
4	1	4
2	2	4

TEACHER RESOURCE MATERIALS

SPACES
Measuring in Metric
From Here to There with Cuisenaire Rods - Area, Perimeter and Volume
Dot Paper Geometry
Let's Pattern Block It

XIV. TIME

Objective 36 Determine elapsed time.

APPROPRIATE MATERIALS

Clocks, digital and analog
Sand timers
Stopwatches
Calendars
Timetables and schedules

ENABLING SKILLS AND ACTIVITIES

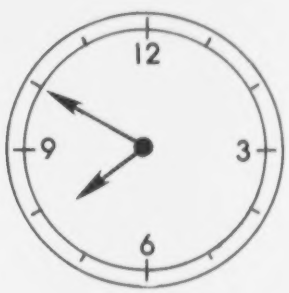
Telling time from a clock face or digital display is a skill that is necessary for measuring time. The time read from a clock serves as a reference point; it indicates what time it is now. In the practical application of time, we usually compare the present time to the time a past event has happened or to the time a future event will occur.

The sequencing of events is an important concept. Students must have opportunities to sequence events (before, during, after, much later, and so on) and link the sequence to the time each event occurs.

The duration of an event is another important time concept. Students should estimate and measure "about how long it takes" to do everyday tasks, like walk to school, wash all the chalkboards, or deliver 20 newspapers. Have students list the tasks that might take 5 minutes, 15 minutes, 30 minutes, an hour, or a day. Do some of the tasks and time them with a sand timer. Time the tasks with a stopwatch and discuss which measuring device is more accurate.

After measuring elapsed time directly, engage in activities that measure duration of time indirectly. Record the time a task is started and the time the task is completed. How can we find the elapsed time? One efficient way, for adults as well as students, is to count the hours or minutes. For example, if a task starts at 9:50 A.M. and is completed at 10:25, the elapsed time may be counted in minute or five-minute intervals--9:55, 10:00, 10:05, 10:10, 10:15, 10:20, 10:25. There are seven intervals of five minutes each--the task took 35 minutes. Allow students to use the counting strategy. They will begin to make it more efficient; the same example will later be reorganized into a 10-minute and a 25-minute interval. More efficient strategies will evolve naturally--you do not need to suggest them.

As students work on elapsed time problems, they should have clock faces and digital clocks available. They need to establish visual methods to quickly identify intervals of time. If a clock face looks like this



and we know we have to leave the house at 8:05 A.M. to catch the bus, we can visualize the interval by imagining where the minute hand will be and comparing the part of the circle between the two positions of the minute hand (now and at 8:05) to the entire circle. One-fourth of the circle is one-fourth of an hour. We have 15 minutes to finish getting ready before we must leave the house.

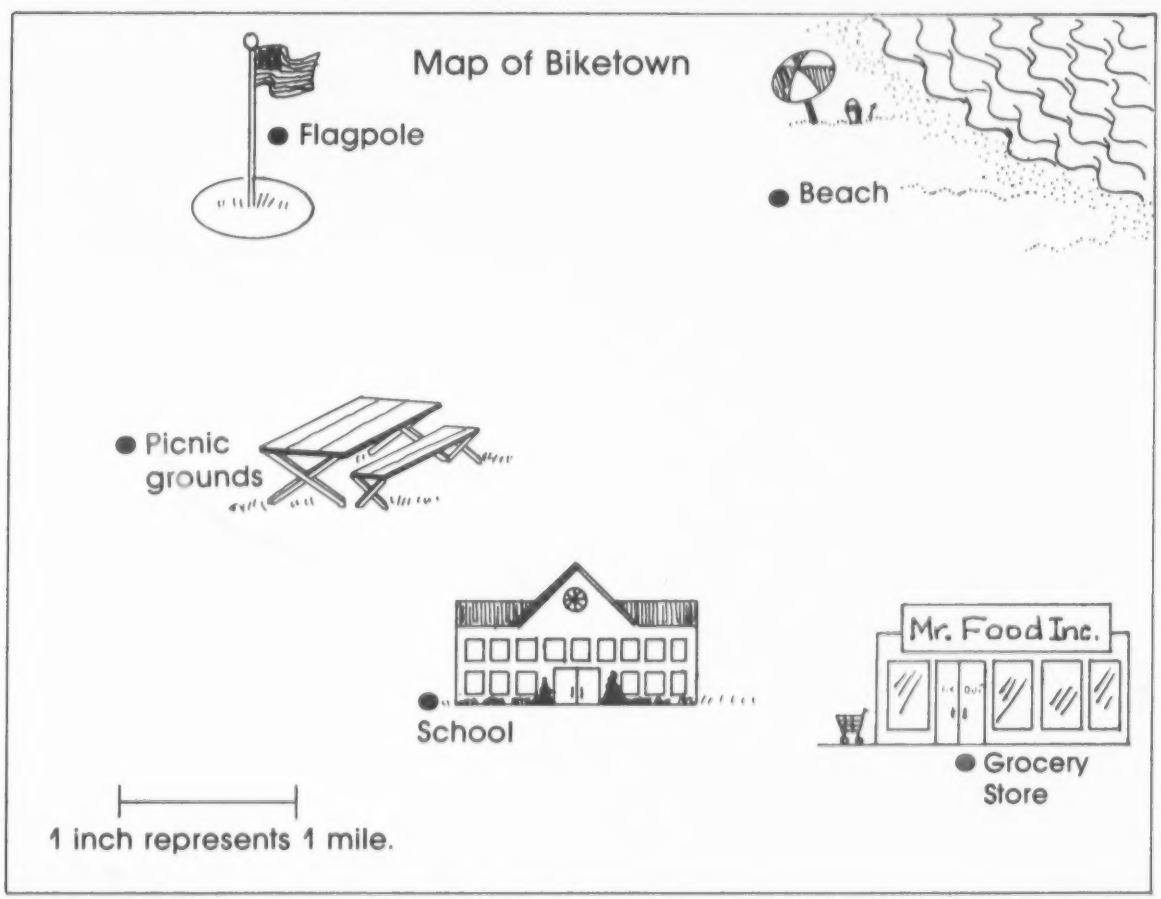
Provide a variety of activities to develop time concepts. Sequence events, measure time with various instruments, estimate the duration of tasks, and use time as a reference point to indirectly measure elapsed time. Magazines, newspapers, history and geography books, bus and train schedules, and everyday classroom occurrences are all resources that can be used.

SAMPLE LESSON Elapsed Time Problems from a Map Reading Activity

MATERIALS NEEDED

A map
Strips of paper
Pencils

Start with a simple map, perhaps using landmarks in your own town. We will be taking a bicycle trip around the town. The average speed of the bicycles will be 10 miles per hour. The scale on the map shows that one inch represents one mile.



Decide on a starting time and a route. At each landmark we will rest for ten minutes. The task is to list the arrival and departure times for each landmark.

Ask other questions about the trip. Will we be able to complete the route in one day? What if our average speed is five miles per hour?

Change the route and repeat the activity. Try another map, even a real one of the town. Take a trip by car at 30 miles per hour. Explore other areas--the state, the country. Decide on a mode of transportation and an average speed. How long will it take to get from Chicago to New Haven? Incorporate the activity into your geography lesson and explore travel times between foreign cities--Paris to Rome, perhaps.

TEACHER RESOURCE MATERIALS

Measurement and the Child's Environment
SPACES
The Good Time Math Event Book
Problem Solving Focus: Time and Money
Thinking About Time

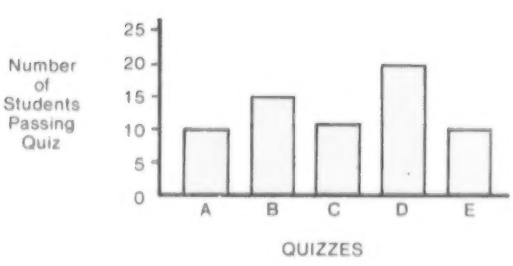
OBJECTIVE	SAMPLE TEST ITEM
CONCEPTUAL UNDERSTANDINGS	
1. Order whole numbers less than 100,000.	Which group of numbers is in order from SMALLEST TO LARGEST? a. 935 1240 827 536 b. 2635 9236 15,212 8014 c. 7239 7356 9236 10,110 d. 6235 1725 920 1836
2. Identify the value of a digit in whole numbers less than 100,000 and rewrite whole numbers using expanded notation.	What is the value of the 5 in the number 35,620? a. 5 b. 5000 c. 500 d. 50,000
3. Rename whole numbers by regrouping 1000's, 100's, 10's, and 1's.	7 thousands, 5 hundreds, and 26 tens equals: a. 7726 b. 7760 c. 7526 d. 7506
4. Round whole numbers less than 100,000 to the nearest 1000, 100, and 10.	What is 5926 when rounded to the nearest thousand? a. 5000 b. 5920 c. 5900 d. 6000

OBJECTIVE	SAMPLE TEST ITEM
CONCEPTUAL UNDERSTANDINGS, continued	
5. Multiply and divide multiples of 10 and 100 by 10 and 100.	90 x 10 = a. 9000 b. 90,000 c. 90 d. 900
6. Identify equivalent fractions and mixed numbers using pictures.	What fractional part of these figures is shaded? a. $2\frac{1}{2}$ b. $2\frac{2}{3}$ c. $1\frac{3}{4}$ d. $1\frac{4}{3}$
7. Identify equivalent fractions and mixed numbers.	Which of the following pairs of fractions is equivalent? a. $\frac{2}{3}, \frac{4}{6}$ b. $\frac{1}{9}, \frac{4}{10}$ c. $\frac{2}{3}, \frac{1}{2}$ d. $\frac{6}{14}, \frac{2}{9}$

OBJECTIVE	SAMPLE TEST ITEM
CONCEPTUAL UNDERSTANDINGS, continued	
8. Identify decimals (.01 to 2.99) from pictorial representations.	The shaded part of this picture shows what decimal number? a. 18.0 b. 1.8 c. 1.2 d. 2.0
9. Extend patterns involving numbers and attributes.	Which number is missing in this pattern? 29, 32, 36, 41, ___, 54, 62 a. 52 b. 49 c. 47 d. 56
10. Identify an appropriate procedure for making estimates for whole number computations.	Which of the following would be a good way to estimate 286 x 406 = a. 300 x 400 = b. 200 x 400 = c. 200 x 500 = d. 300 x 500 =
COMPUTATIONAL SKILLS	
11. Add and subtract 2-, 3-, and 4-digit whole numbers and money amounts less than $100.00.	$79.26 - $24.32 = a. $94.64 b. $23.42 c. $64.94 d. $54.94

OBJECTIVE	SAMPLE TEST ITEM
COMPUTATIONAL SKILLS, continued	
12. Know multiplication and division facts.	9 x 7 a. 35 b. 49 c. 63 d. 56
13. Multiply 2- and 3-digit whole numbers and money amounts less than $10.00 by 1-digit numbers.	$6.42 x 4 a. $25.68 b. $2568 c. $256.80 d. $25,680
14. Divide 2- and 3-digit whole numbers by 1-digit numbers.	$2\overline{)256}$ a. 94 b. 126 c. 123 d. 128
15. Add and subtract fractions and mixed numbers with like denominators (without regrouping mixed numbers).	$5\frac{2}{5}$ $+2\frac{1}{5}$ a. $3\frac{1}{5}$ b. $7\frac{3}{5}$ c. $7\frac{1}{3}$ d. $7\frac{2}{5}$

OBJECTIVE	SAMPLE TEST ITEM
COMPUTATIONAL SKILLS, continued	
16. Add fractions and mixed numbers with like denominators involving regrouping improper fractions to whole numbers or mixed numbers.	$2\frac{3}{5}$ $+4\frac{5}{5}$ a. $7\frac{3}{5}$ b. $7\frac{1}{3}$ c. $6\frac{1}{5}$ d. $7\frac{1}{2}$
17. Add and subtract fractions and mixed numbers with unlike denominators (one denominator a factor of the other).	$\frac{9}{12} - \frac{1}{6} =$ a. $\frac{2}{3}$ b. $\frac{5}{6}$ c. $\frac{7}{12}$ d. $\frac{3}{4}$
18. Find fractional parts of whole numbers.	What is $\frac{1}{8}$ of 32? a. 9 b. 24 c. 4 d. 6
19. Estimate sums and differences of whole numbers and money amounts.	\$900 is a good estimate for which of the following? a. \$527 + \$129 = b. \$437 + \$225 = c. \$632 + \$925 = d. \$237 + \$682 =

OBJECTIVE	SAMPLE TEST ITEM
COMPUTATIONAL SKILLS, continued	
20. Estimate products and quotients of whole numbers and money amounts (1-digit factor and 1-digit whole number divisor).	ABOUT how much is $4\overline{)\$795}$? a. $400 b. $200 c. $300 d. $600
PROBLEM SOLVING AND APPLICATIONS	
21. Interpret graphs, tables, and charts.	 Which two quizzes did the most students pass? a. B and D b. A and E c. C and D d. B and C

OBJECTIVE	SAMPLE TEST ITEM

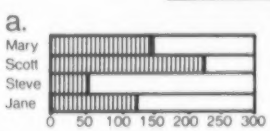
PROBLEM SOLVING AND APPLICATIONS, continued

22. Identify the graph that best illustrates given data.

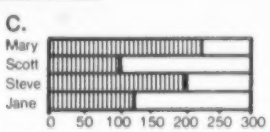
Which of the following graphs correctly shows the information given on the table?

A-1 Used Car Sales	
Mary	150 Used Cars
Scott	225 Used Cars
Steve	50 Used Cars
Jane	125 Used Cars

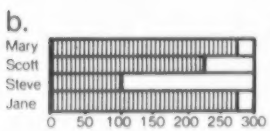
a.



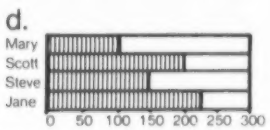
c.



b.



d.



23. Identify number sentences from problems.

Kathy has 265 more pages to read in her book. She has already read 327 pages.

Which number sentence would you use to find out how many pages she will read in all?

a. 592 - 265 =
b. 2(327 + 265) =
c. 592 - 327 =
d. 265 + 327 =

24. Solve 1-step problems involving whole numbers and money amounts.

The employees of the ice cream shop sold 23 quarts of ice cream a month for 9 months. How many quarts of ice cream did they sell in all?

a. 32
b. 239
c. 207
d. 113

OBJECTIVE	SAMPLE TEST ITEM
25. Solve problems involving making change.	If you buy two candy bars at $.15 apiece, how much money will you have left from a $1.00 bill? a. $0.85 b. $0.70 c. $0.50 d. $0.65
26. Solve 1-step problems involving fractions.	A painter bought $7\frac{1}{2}$ quarts of flat paint, and $2\frac{3}{4}$ quarts of enamel paint. How many quarts of paint does he have? a. $12\frac{1}{2}$ b. $11\frac{2}{3}$ c. 10 d. $10\frac{1}{4}$
27. Solve 2-step problems involving whole numbers and money amounts.	If you bought eight tickets at $3.00 apiece, and a T-shirt for $7.50, how much money would you spend? a. $10.50 b. $24.00 c. $31.50 d. $16.50
28. Estimate a reasonable answer to a given problem.	The plant shop owners are having a sale on marigolds. They are selling them at $1.96 a plant. ABOUT how many plants could be bought with $20.00? a. 20 b. 10 c. 38 d. 15

OBJECTIVE	SAMPLE TEST ITEM

PROBLEM SOLVING AND APPLICATION, continued

29.	Identify extraneous information in problems and solve problems with extraneous information.	The ice cream shop started the day with 27 quarts of vanilla, 32 quarts of chocolate, and 30 quarts of strawberry ice cream for the grand opening sale. During the first hour 12 quarts of vanilla, 18 of chocolate, and 22 of strawberry ice cream were sold. How much chocolate ice cream is left? a. 14 quarts b. 37 quarts c. 89 quarts d. 52 quarts
30.	Identify needed information in problem situations.	Mary needs to be at the bus station by 3:00 P.M. to catch her bus to Medina. She left home by car at 1:30 P.M. What else do you need to know to find out if she will arrive in time to catch her bus? a. How fast the bus goes. b. How long the car ride will take. c. The time the bus arrives in Medina. d. How long the bus ride will take.
31.	Solve process problems involving the organization of data.	Scott ate 15 cookies more than Sue. Kim ate 3 less than Sue ate. If Kim ate 21 cookies, then how many cookies did Scott eat? a. 39 b. 24 c. 21 d. 27

OBJECTIVE	SAMPLE TEST ITEM

MEASUREMENT AND GEOMETRY

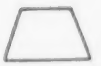
32. Identify geometric figures.

Which of the following figures is a rectangle?

a.

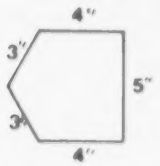
c.

b.

d. 

33. Measure/determine perimeters and areas.

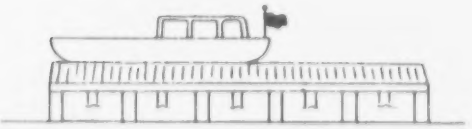
What is the perimeter of this figure?



a. 48"
b. 20"
c. 15"
d. 19"

34. Estimate lengths and areas.

If the boat is 15 feet long, ABOUT how long is the dock?



a. 56 ft.
b. 12 ft.
c. 25 ft.
d. 32 ft.

OBJECTIVE	SAMPLE TEST ITEM
MEASUREMENT AND GEOMETRY, continued	
35. Select appropriate metric or customary units and measures.	Which of the following is best for measuring the length of a soccer field? a. Meters b. Centimeters c. Kilometers d. Milliliters
36. Determine elapsed time.	What time is it 2 $\frac{3}{4}$ hours after 7:30 P.M.? a. 9:15 P.M. b. 9:30 P.M. c. 10:15 P.M. d. 10:30 P.M.

APPENDIX B
SUPPLIERS OF COMMERCIAL MATERIALS

Supplier	Materials
Burt Harrison & Co. P.O. Box 732 Weston, MA 02193-0732 (617) 647-0674	Manipulatives
Creative Publications 5005 West 110th Street Oak Lawn, IL 60453 (800) 624-0822	Manipulatives Resource Materials
Cuisenaire Co. 12 Church Street New Rochelle, NY 10805 (914) 235-0900	Manipulatives Resource materials
Dale Seymour Publications P.O. Box 10888 Palo Alto, CA 94303 (800) 872-1100	Resource materials
Didax 6 Doulton Place Peabody, MA 01960 (617) 535-4757	Manipulatives Resource materials
National Council of Teachers of Mathematics 1906 Association Drive Reston, VA 22091 (703) 620-9840	Professional publications Resource materials

Note: The above list includes suppliers of materials mentioned in this handbook. The list is NOT exhaustive, but does include many regional suppliers. Your school office may have their catalogs, as well as those of other suppliers. The addresses provided here are for your convenience in acquiring information.

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